Compressive Sampling with R: A Tutorial

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data analysis that delivers

15 JUNE 2011

Plan

- Analog-to-Digital conversion: Shannon-Nyquist Rate
- Medical Imaging to One Pixel Camera
- Compressive Sampling Frame Work
- CS via Convex Programming
- Doing CS with R

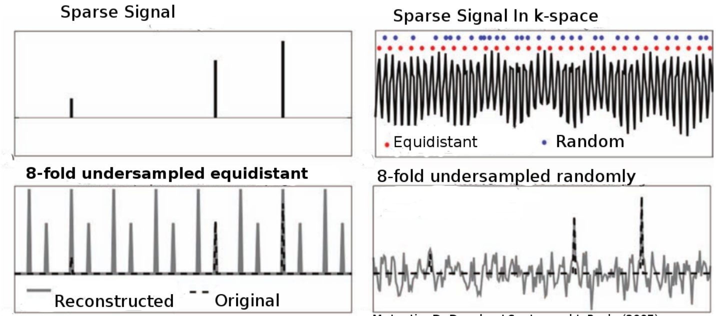
Analog-to-Digital conversion: Shannon-Nyquist Rate

1

- A time-varying bandwidth limited signal with no frequencies higher than N hertz can be perfectly reconstructed by sampling the signal at regular intervals of $1/2N$ seconds. [2]
- Converse Argument
  A signal with frequencies higher than N hertz *cannot* be reconstructed uniquely by sampling the signal at regular intervals of 1/2N seconds (aliasing).
- New Argument
  But reconstruction of the signal (image) is possible with *random under-sampling*, with **Compressive Sampling** [3] methodology if image (information) is sparse or compressible. (Example?)

[1] AMS What's Happening in the Mathematical Sciences, Vol. 7, 2009
[2] C. E. Shannon 1949 and H. Nyquist 1928
[3] Donoho 2006 and Candès-Romberg-Tao 2006

What is Compressive Sampling about?

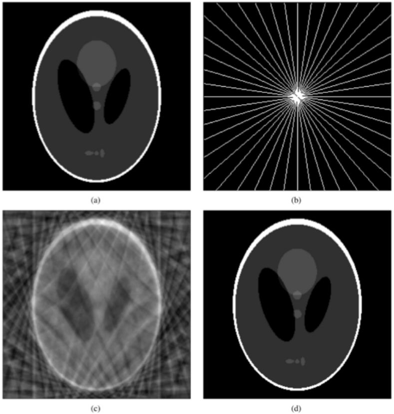


[4] M. Lustig, D. Donoho, J.Santos and J. Pauly (2007)

- Sparsity: "k-sparse" signals
- Randomness: random sampling with K log(N/K) samples. (N being the size of the signal)
- Historical Examples !

[4] Lustig M. et al (2007)

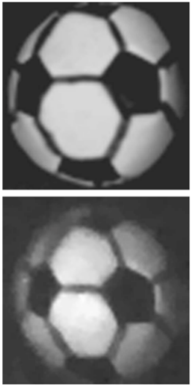
Medical Imaging



Sampling rate: About almost 50 times smaller than the Nyquist rate!
(Implies faster acquisition times.) [5]

[5] Candes - Romberg - Tao, IEEE transactions information theory (2007) (Tao here is Terence Tao, 2006 Fields Medalist!)

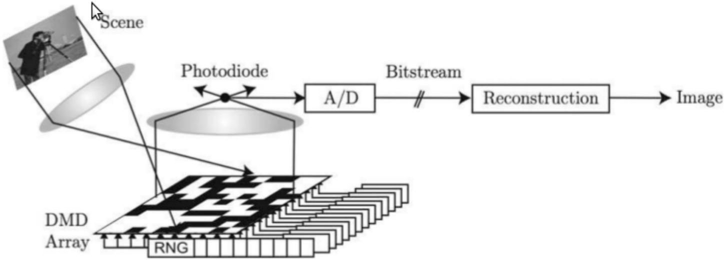
One Pixel Camera


[6]

- Upper 64 x 64 pixel, Lower 1 pixel camera with 1600 measurements

[6] R Baraniuk et al (2007)

One Pixel Camera

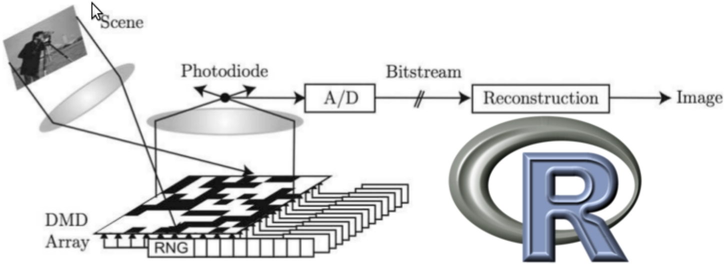


[7]

- 1 pixel camera. Where can we use R?

[7] R Baraniuk et al (2007)

One Pixel Camera

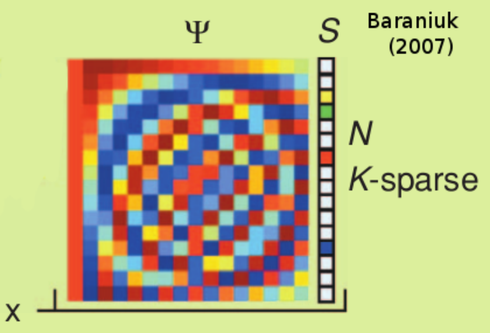


[8]

- **R1magic** R package on incoming CRAN!
- Some linear algebra!

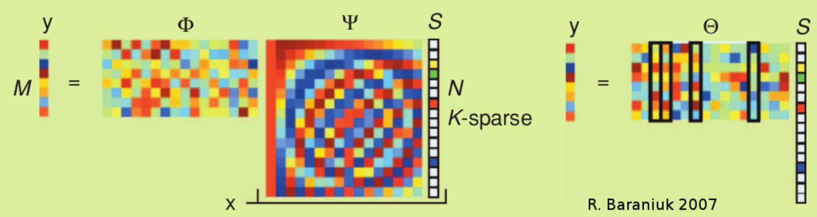
[8] R Baraniuk et al (2007)

Transformation for Sparsification

- The signal (image) x may have $K-$sparse representation, a vector S in another domain (orthonormal basis).
- $$x = \Psi S$$
- Ψ can be any orthonormal transformation (Fourier, Wavelet, Curvelet etc.)
- 

Compressed Sensing (CS) Framework: ℓ_1 minimization

[9] [10]



- The matrix Φ must be incoherent with respect to Ψ (uncorrelated bases).
- $\Theta = \Phi\Psi$ is called *CS-matrix*.
- Solution to the problem: ℓ_1 constrained minimization : $min||\Psi S||_1$ s.t. $\Phi\Psi S = \mathbf{y}$
- ℓ_1-regularized least-squares $min\left(||\Phi\Psi S - y||_2^2 + \lambda||\Psi S||_1\right)$ with λ, regularization parameter.

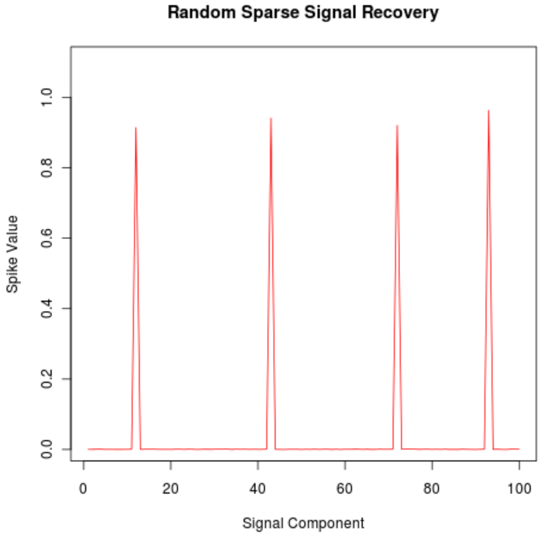
[9] Donoho 2006 and Candès-Romberg-Tao 2006
[10] R. Baraniuk, 2007

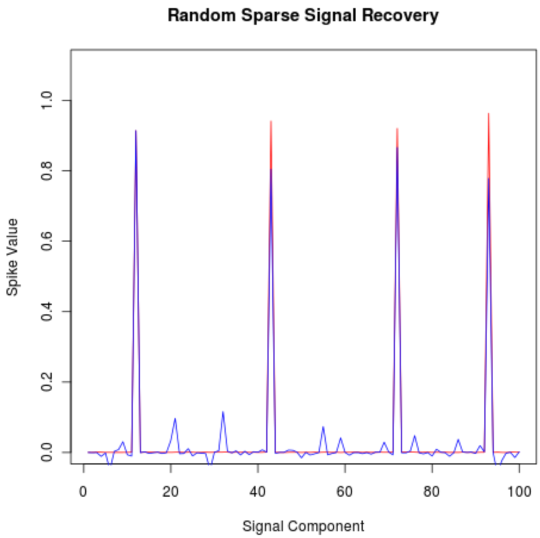
R1magic provides Basic Tools for CS

- Random sparse signal generation.
- l1, l2 and TV constrained minimizations.
- Random measurement matrix generation.
- Bases matrices.
- Automated penalty parameter selection (TODO).
- Advanced re-weighted minimization to enhance sparsity (TODO).

Demonstration with simple example with random data:

```
library(R1magic)
N <- 100 ;#  Signal
K <- 4  ;#  Sparsity
;#  Up to Measurements  > K LOG (N/K)
M <- 40
;# Measurement Matrix (Random Sampling Sampling)
phi <- GaussianMatrix(N,M)
;# R1magic generate random signal
xorg <- sparseSignal(N, K, nlev=1e-3)
y <- phi %*% xorg ;# generate measurement
T <- diag(N) ;# Do identity transform
p <- matrix(0, N, 1) ;# initial guess
;# R1magic Convex Minimization !
;# (unoptimized penalty parameter)
ll <- solveL1(phi, y, T, p)
x1 <- ll$estimate ;# Returns nlm object
```


Random Sparse Signal Recovery
Spike Value
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
Signal Component


Random Sparse Signal Recovery
Spike Value
0.0
0.2
0.4
0.6
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1.0
0
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Thank You!